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Analysis of Robust Stability and Performance for Two-Degree of Freedom Control Structure with Dynamic Controller

Sanghwa Jeong*

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In the two-degree of freedom control, the performance of good command following and disturbance rejection are considered separately. Qualitatively, good performance is equivalent to minimizing the energy of the error for any inputs. In this work, using H_∞-formulation in the frequency domain, robust stability and robust performance specifications have been analyzed for the two-degree of freedom control structure with a dynamic controller. When the two-degree of freedom system having a feed-forward loop is controlled by a dynamic controller, two different performance weight functions are imposed and the robust performance specification is proposed in terms of the return ratio and feed-forward loop. The design algorithm in the frequency domain is illustrated for the simplified retail model of Industrial Dynamics to compare three kinds of control laws, which are the output feedback control scheme and two additional dynamic control ones. Numerical simulation results show that the dynamic control laws provide a larger robust stability margin than the output feeback control one and has good performance robustness for disturbance rejection at low frequencies.

Key Words: Dynamic Controller, Two-Degree of Freedom Control Structure, H_∞ Control, Stability Robustness, Performance Robustness

Nomenclature

IESF : Integrated-Error with State-Feedback controller
ISFF : Integrated-Error with State-Feedback and Filtering controller
H_∞ : Hardy space
$\varepsilon_j(s)$: Sensitivity transfer function in the s-domain for $j=1, 2, 3$
$\eta_j(s)$: Complementary sensitivity transfer function in the s-domain for $j=1, 2, 3$
$\mu_j(s)$: Pseudo-sensitivity transfer function in the s-domain for $j=1, 2, 3$
subscript 1, 2, 3 : 1 stands for output feedback, 2 for IESF, and 3 for ISFF
$w(s)$: Performance weighting function
l_m : Multiplicative uncertainty
Δ : Uncertainty
DUD : Delay due to unfilled orders at distributor
UOD : Unfilled orders at distributor
IAR : Actual inventory at retailer

* Factory Automation Research Center for parts of vehicles, Dept. of Mechanical Engineering, Chosun University, Kwangju, Korea.

1. Introduction

Although the goal of a controller design is typically based upon its time-domain response, robust control analysis is often carried out in the frequency-domain because it gives a more convenient information of describing model uncertainty. Also, one can define easily the desired performance in the frequency-domain. The robust controller design methods in the frequency domain are described by Maciejowski (1989) and Morari (1989). In the frequency domain, robustness is measured either as gain and phase margins or the tolerance of plant perturbations. A detailed account of these methods for continuous-time systems can be found in Maciejowski (1989). When dealing with robust performance in the context of linear feedback systems with H_∞-norm performance objectives, Doyle (1982) introduced a measure of performance for linear feedback systems in the presense of structured model uncertainties. This approach is based on a matrix function called the structured singular value, where stability and performance robustness are dealt with in the same framework. Recently, a new method is presented for defining the bounds on achievable robust performance for linear systems by Morari and his associates(Laughlin, et al., 1986; Lewin, 1988, 1991; Rivera, 1987, 1992; Zafiriou, 1991). Using the H_∞-formulation, performance to the model reduction problem of internal model control proposed by Morari (1989).

The general configuration of feedback control system is shown in Fig. 1. Note that r is the reference input, d is the disturbance, and n is the measurement noise. If the restrictions $K=I$ and $F=I$ are imposed on the controller, the system has only one-degree of freedom. The closed loop system tracks the command signals in the same way as the measurement noise. Performance for disturbance rejection and command following cannot be influenced independently when the error is defined as $e=r-y$. However, when the feedback part of controller K and the prefilter F have been designed, the performance for disturbance rejection and command following can be achieved independently. The independence between command following and disturbance rejection is the merit of the complete closed-loop system, which is called by a two-degree of freedom control system (Lunze, 1989). This type of system has frequently appeared in the manufacturing production-distrubution systems. Morari (1989) has developed robust stability and performance specification for two-degree of freedom system with internal model control (IMC), which has a desired process model. If a controller has a n-th order error dynamics, it is referred to the dynamic controller. In the design of control systems, such as tracking systems, it is necessary to eliminate completely the effect of offset errors caused by bounded disturbances. Integral action on the dynamic controller results in the closed-loop system in which the outputs follow step commands and reject unmeasurable arbitrary disturbances with bounded constant values. General formulations of two kinds of dynamic controller and robust stability for the dynamically controlled systems have been developed by Jeong (1992, 1993, 1994).

In this work, using H_∞-formulation in the frequency domain, robust stability and robust performance specification have been applied for the two-degree of freedom control system controlled by the dynamic controllers. Two different performance weight functions are imposed to analyze the two-degree of freedom control system with uncertainty.

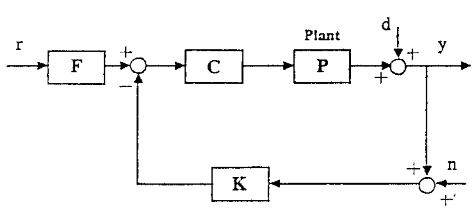


Fig. 1 General configuration of a feedback control structure

2. System Description of Two-Degree of Freedom Control

If r and d behave differently, then additional controller blocks, which are generally pre-filters $f(s)$ and feedback blocks $k(s)$, are required to allow independent adjustments for both r and d. This type of system is referred to as a two-degree of freedom control (Morari, 1989; Lunze, 1989) as shown in Fig. 1. It is assumed that $p(s)$, $c(s)$, and $k(s)$ are real-rational and proper, with at least one of them strictly proper. Measurement noise will be neglected globally in this paper.

The relationship between the inputs and the error $e(s)=r-y$ for the two-degree of freedom control system can be expressed by

$$e(s)=\varepsilon(s)d+\mu(s)r \quad (1)$$

The relationships between the inputs and outputs are given by

$$\varepsilon(s)=\frac{e}{d}=\frac{-1}{1+p(s)c(s)k(s)} \quad (2)$$

$$\mu(s)=\frac{e}{r}=1-\frac{p(s)c(s)f(s)}{1+p(s)c(s)k(s)} \quad (3)$$

where $\varepsilon(s)$ is defined by a sensitivity function and $\mu(s)$ is defined by a pseudo-sensitivity function.

In this system, $c(s)$ and $k(s)$ can be designed first for disturbance rejection and then the prefilter $f(s)$ can be selected independently for good command following. The independence of those two performance measures is the merit of the two-degree of freedom control system. This two-degree of freedom control frequently appears in manufacturing process control problems. For the constant feedback block $k(s)$, three kinds of control laws for $c(s)$ are investigated in this paper: output feedback and two kinds of dynamic control laws (IESF and ISFF). The block diagrams of those control schemes for the two-degree of freedom control structure are shown in Fig. 2, Fig. 3, and Fig. 4.

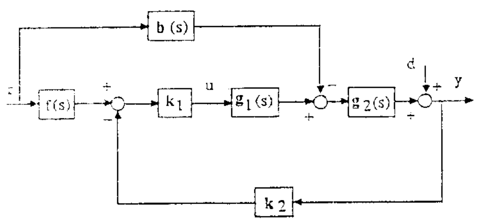


Fig. 2 Two-degree of freedom system with output feedback control law

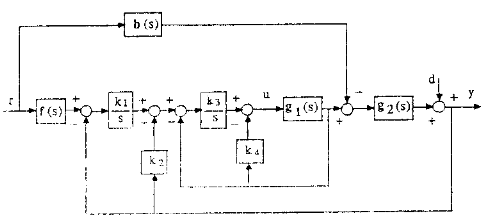


Fig. 3 Two-degree of freedom system with IESF control law

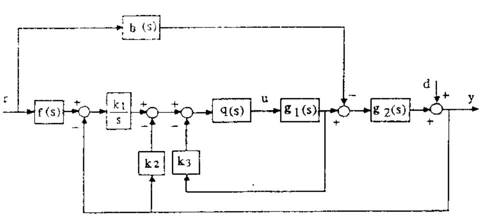


Fig. 4 Two-degree of freedom system with ISFF control law

3. Nominal H_∞ Performance Specifications

Consider a single-input single-output (SISO) two-degree of freedom control system. In general, performance objectives are good command following and disturbance rejection, the maximum error should be made as small as possible for each input. In the two-degree of freedom control, one has to consider good command following performance and disturbance rejection separately. Qualitatively, good performance (or response) is equivalent to minimizing the energy of the error e for any inputs r and d. Hence performance requirements are imposed by placing a bound on the magnitudes of the sensitivity function $\varepsilon(jw)$ and pseudo-sensitivity function $\mu(jw)$.

First, consider the performance for disturbance rejection. If the performance weighting function for disturbance rejection $w_d(s)$ is chosen to satisfy the characteristics of the disturbance d to be rejected, then the performance specification is met as follows (Morari, 1989):

$$|\varepsilon(jw)| \le |w_d|^{-1}, \ \forall w \tag{4}$$

where $|w_d|^{-1}$ represents an upper bound on the sensitivity function $\varepsilon(jw)$ and the weighting function determines the shape of the sensitivity function $\varepsilon(jw)$, i.e., its relative magnitude at different frequencies. If a controller is chosen such that

$$\|\varepsilon_i w_d\|_\infty = \sup_w |\varepsilon_i w_d| < 1, \ \forall w \tag{5}$$

then, the performance specification (4) is always satisfied (Morari, 1989; Freudenberg, 1986). The subscript $i = 1, 2, 3$ stands for the output feedback, IESF and ISFF control respectively.

Next, applying the above procedure for good command following performance, the following condition can be derived:

$$\|\mu_i w_r\|_\infty = \sup_w |\mu_i w_r| < 1, \ \forall w \tag{6}$$

where w_r is a reference input weighting function. From Eqs. (5) and (6), the controllers $c(s)$ and $k(s)$ can be selected for disturbance rejection first and then one prefilter $f(s)$ can be chosen for good command following independently.

4. Stability and Performance Robustness

Multiplicative unstructured uncertainty in a nominal plant can be accounted for in the control system design because of uncertainty in the actual process parameter and because the linearized plant may be changed as the operating point changes. One considers a set of actual plants $\tilde{p}(jw)$ with a multiplicative uncertainty l_m as follows (Morari, 1989 ;, Lewin, 1991) :

$$\tilde{p}(jw) = p(jw)(1 + l_m(jw)) \quad |l_m| < l_M(w),\ \forall w \tag{7}$$

The set of all possible plant models in the complex plant is defined by

$$\Pi(w) = \{\tilde{p}(s) : |\tilde{p}(jw) - p(jw)| \le l_M \ \forall w\} \tag{8}$$

where $p(jw)$ is a nominal plant, and

$l_M(w)$ is an upper-bound on multiplicative uncertainty $l_m(jw)$.

$l_M(w)$ is equivalent to representing process uncertainty by a disc-shaped uncertainty region $\Pi(w)$ with radius $|p(jw)|\,l_M(w)$, centered on $p(jw)$ (Laughlin, et. al., 1986). The system is closed-loop stable if the net number of counter clockwise encirclements of the point $(-1, 0)$ by $pc(s)$ as s traverses the Nyquist contour is equal to the number of unstable open-loop poles.

4.1 Stability robustness

Consider a one-degree of freedom control. Robust stability of plants defined by Eqs. (7) and (8) can be derived by using the Nyquist stability criterion. Assuming that the nominal closed loop plant $p(jw)$ is stable, the system with multiplicative uncertainty $l_m(jw)$ upper-bounded by $l_M(w)$ is robust stable with a specific controller $c(s)$ if and only if the distance of pc from the point $(-1, 0)$, which is $|1 + pc(jw)|$, exceeds a disc-radius $|pc(jw)|\,l_M(w)$. Hence, the robust stability condition is given by $|1 + pc(jw)| > |pc(jw)|\,l_M(w)$, $\forall w$, or equivalently, $|\eta(jw)|\,l_M(w) < 1$, $\forall \omega$, where $\eta = pc(1 + pc)^{-1}$; the nominal complementary sensitivity function. Rewriting the above inequality, one finds (Morari, 1989) ; $\|\eta l_M\|_\infty \equiv \sup_w |\eta l_M| < 1$. This condition is a special case of small gain theorem where a bounded input produces a bounded output and is not only sufficient but also necessary for robust stability.

In two-degree of freedom control, the above analysis can be appled. Because a feedforward transfer function $b(s)$ and a prefilter $f(s)$ are not in the feedback loop, they have no effect on stability only if $b(s)$ and $f(s)$ are stable. If the uncertain process $\tilde{g}_1(s)$ shown in Fig. 2, Fig. 3, and Fig. 4 is associated with the nominal process $g_1(s)$ expressed by

$$\tilde{g}_1(s) = (1 + l_m)g_1(s),\ |l_m(jw)| < l_M(w) \quad \forall w \tag{9}$$

then, robust stability depends only on

$$\|\eta_i l_M\|_\infty = \sup_w |\eta_i l_M| < 1 \quad \text{for } i = 1, 2, 3 \tag{10}$$

The subscript $i = 1, 2, 3$ stands for the output feedback, IESF and ISFF control respectively.

In the H_∞ control structure of the two-degree of freedom control, $\|\varepsilon_i w_d\|_\infty$ for the performance of disturbance rejection, $\|\mu_i w_r\|_\infty$ for the performance of good command following, and $\|\eta_i l_M\|_\infty$ for robust stability should be small. The performance of disturbance rejection and the robust stability give rise to a trade-off due to the relation of $\varepsilon(s)$ and $\eta(s)$. That means making one small will make the other one large. Hence in this case, the best compromise between the conflicting objectives of performance of disturbance rejection and stability robustness should be reached in the robust control system and then prefilter $f(s)$ should be designed for the performance of good command following.

4.2 Performance robustness

The following geometric relation can be obtained as follows (Morari, 1989) :

$$|1 + pc| - |1 + \tilde{p}c| \le |pc|\,l_M \quad \forall \tilde{p} \in \Pi \tag{11}$$

or equivalently,

$$|\tilde{\varepsilon}| = \left|\frac{1}{1 + \tilde{p}c}\right| \le \frac{|\varepsilon|}{1 - |\eta|\,l_M} \quad \forall \tilde{p} \in \Pi \tag{12}$$

In the two-degree of freedom control, the robust performance specification for disturbance rejection is the same type of Eq. (5) for the nominal performance specification and is given by

$$\| \tilde{\varepsilon}_i w_d \|_\infty = \sup_w | \tilde{\varepsilon}_i w_d | < 1 \tag{13}$$

Substituting Eq. (12) into Eq. (13), the robust performance specification for disturbance rejection can be rewritten in terms of the nominal plant functions as

$$| \varepsilon_i w_d | + | \eta_i l_M | < 1, \ \forall w \tag{14}$$

The first term implies nominal performance of Eq. (5) and the second one implies robust stability of Eq. (10). In other words, if the condition Eq. (14) is satisfied, one guarantees both robust stability and nominal performance for disturbance rejection. Hence Eq. (14) represents the robust performance specification for disturbance rejection in the H_∞ control framework. Based on the robust performance specification Eq. (14) for disturbance rejection, any types of controllers can be designed to satisfy this performance specification.

Next, consider a robust performance for good command following from the error relationship at each control. In a similar manner, the robust performance specification for good command following is given by

$$\| \tilde{\mu}_i w_r \|_\infty = \sup_w | \tilde{\mu}_i w_r | < 1, \ \forall w \tag{15}$$

Let the nominal pseudo-sensitivity function μ_i in the given two-degree of freedom system be defined by

$$\mu_i = \frac{\beta_{1i} - \beta_{2i}}{1 + \alpha_i} - k_0, \ for \ i = 1, 2, 3 \tag{16}$$

where k_0 is the overall constant gain between the reference input and the output, β_{1i} is a feed-forward loop between the reference input and the output, β_{2i} is another feed-forward loop between the reference input and the output not associated with a feedback loop, and α_i is a return ratio (MacFarlane, 1970). It is assumed that β_{2i} is independent on the uncertainty l_{mi}. Then the pseudo-sensitivity function with an uncertainty l_{mi} in the plant can be written as follows

$$\tilde{\mu}_i = \frac{\beta_{1i}(1 + l_{mi}) - \beta_{2i}}{1 + \alpha_i(1 + l_{mi})} - k_0 \quad for \ i = 1, 2, 3 \tag{17}$$

Substituting Eq. (17) into Eq. (15), one obtains

$$\left| \left(\frac{\beta_{1i}(1 + l_{mi}) - \beta_{2i}}{1 + \alpha_i(1 + l_{mi})} - k_0 \right) w_r \right| < 1 \quad \forall w \tag{18}$$

It is noted that the nominal functions are

$$\mu_i = \frac{\beta_{1i} - \beta_{2i}}{1 + \alpha_i} - k_0 \ \text{ and } \ \eta_i = \frac{\alpha_i}{1 + \alpha_i}$$

Rearranging Eq. (18) in terms of the nominal plant functions, μ_i and η_i, and substituting the upper bound l_M into Eq. (18),

$$\left| \mu_i w_r + \eta_i l_M \left(\frac{\beta_{1i}}{\alpha_i} - k_0 \right) w_r \right| + | \eta_i l_M | < 1 \tag{19}$$

$$\Leftarrow | \mu_i w_r | + | \eta_i l_M | \cdot \left\{ 1 + \left| \left(\frac{\beta_{1i}}{\alpha_i} - k_0 \right) w_r \right| \right\} < 1 \tag{20}$$

This expression is only a sufficient condition for the robust performance of good command following. The first term represents the nominal performance of good command following and the second term is propotional to the robust stability measure $| \eta_i l_M |$. From Eqs. (14) and (20), it can be seen that robust performance specification is achieved if the nominal performance and robust stability are satisfied with some margin. This frequency domain analysis is very complex if the dynamic controller is designed in the time-domain basis. However, those two robust performance specifications are very helpful in the design of the dynamic controller in the freguency domain.

5. Illustrations

Frequency domain analysis is examined at the simplified retail sector based on the SISO H_∞-formulation of stability and performance for the two-degree of freedom control structure. In this example, the reference input r is customer demands received at retailer, the control input u is purchasing rate decisions at retailer, and the output y is actual inventory at retailer. The performance weighting functions, $w_r(s)$ and w_d

(s) are chosen as follows :

$$w_r(s)=\frac{(5s+1)/9}{5s} \text{ : for good command following,}$$

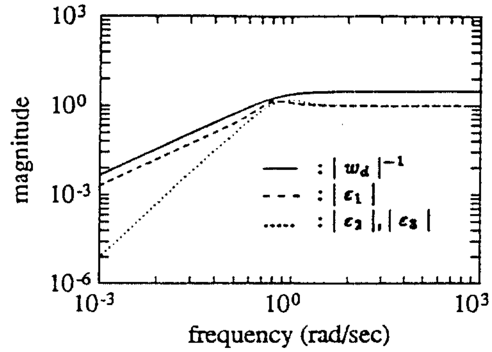
$$w_d(s)=\frac{(1.5s+1)/3}{1.5s} \text{ : for disturbance rejection.}$$

The inverses of $|w_r(jw)|$ and $|w_d(jw)|$ are upper bounds on the magnitude of $|\varepsilon_i(jw)|$ and $|\mu_i(jw)|$. Clearly, this bound constrains $|\varepsilon_i(jw)|$ (or $|\mu_i(jw)|$) (for $i=1, 2, 3$: 1 stands for output feedback, 2 for IESF, and 3 for isff) to be small at low frequencies, since offset-free response is required at that range. In this example, the simplified retail sector as a plant is given as follows (Jeong, 1992 ; Forrester, 1961) :

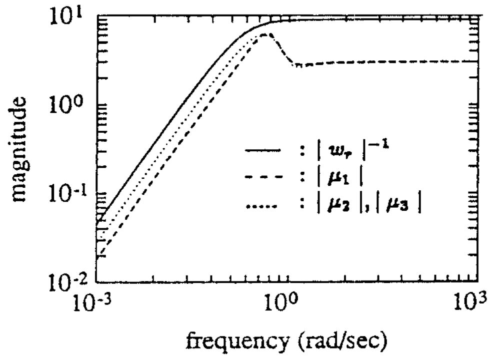
$$g_1(s)=\frac{1}{2s+1},\ g_2(s)=\frac{1}{s}$$

$$b(s)=\frac{1}{s+1},\ f(s)=\frac{3}{2s+1}$$

Here the maximum peaks of the sensitivity functions are selected to be equal (see Fig. 5) for three systems in the nominal plant as a reference to illustrate robustness for the perturbed systems controlled by three kinds of controllers. Gains are selected to satisfy the above design constraint for each controller.



(a) Nominal disturbance rejection



(b) Nominal good command following

Fig. 5 Nominal performance illustration for output feedback, IESF, and ISFF control law : Subscript 1 stands for output feedback, 2 for IESF, and 3 for ISFF.

5.1 Nominal performance

Nominal performance specification of output feedback and dynamic control laws are given by $|\varepsilon_i|<|w_d|^{-1}$ for disturbance rejection. The sensitivity function of the simplified retail sector for each control scheme is expressed as follows :

$$\varepsilon_1(s)=\frac{2s(2s+1)}{4s^2+2s+1} \text{ : for output feedback}$$

$$\varepsilon_{2,3}(s)=\frac{s^2(2s^2+7.04s+9.279)}{2s^4+7.04s^3+9.279s^2+5.427s+1.189} \text{ : for IESF and ISFF}$$

Since the same poles are assigned for the characteristic equation of each dynamic controller, $\varepsilon_2(jw)$ and $\varepsilon_3(jw)$ are same. Fig. 5(a) illustrates the nominal performance for disturbance rejection. The performance specification $|\varepsilon_i(jw)|<|w_d|^{-1}$ of Eq. (4) is satisfied in the three control laws. The dynamic controllers (IESF and ISFF) have better performance for disturbance rejection at low frequencies. Nominal performance specification of good command following is given by $|\mu_i|<|w_r|^{-1}$ from Eq. (6). The pseudo-sensitivity function μ_i of the simplified retail sector for each control scheme is described by

$$\mu_1(s)=\frac{-8s^2-3s+3}{8s^4+16s^3+12s^2+5s+1}-3 \text{ : for output feedback}$$

$$\mu_{2,3}(s)=\frac{-4s^4-16.08s^3-25.598s^2-5.713s+3.566}{4s^6+20.08s^5+41.678s^4+45.731s^3+27.938s^2+8.993s+1.189}-3 \text{ : for IESF and ISFF}$$

where $\mu_2(jw)$ and $\mu_3(jw)$ are same, since the same poles are assigned to the closed-loop characteristic equation. Fig. 5(b) shows the nominal performance for good command followng. Performance specification $|\mu_i|<|w_r|^{-1}$ is satisfied in each control law. The output feedback control law has better performance for good command following at low frequencies. From Fig. 5(a) and (b), one notes that the three control laws have the same performance at high frequencies.

5.2 Stability robustness

The robust stability condition $\| \eta l_{mi} \|_\infty < 1$ are examined to check stability robustness. In this example, it is assumed that the uncertain process $\tilde{g}_1(s)$ is given by

$$\tilde{g}_1(s) = \frac{1}{2.5s+1}$$

The corresponding multiplicative uncertainty l_{mi} due to the uncertain process $\tilde{g}_1(s)$ becomes

$$l_{m1}(s) = \frac{-0.5s}{2.5s+1}$$

$$l_{m2}(s) = \frac{-0.5s^2}{2.5s^2+7.04s+9.279}$$

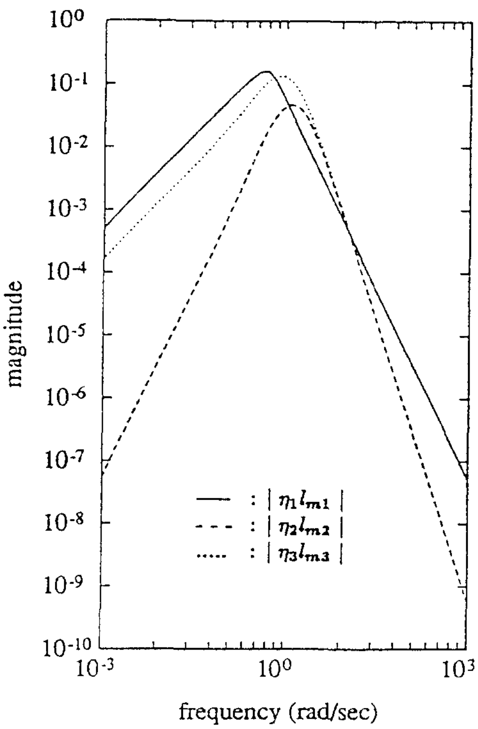


Fig. 6 Magnitudes of $|\eta_i l_{mi}|$ for stability robustness: Subscript 1 stands for output feedback, 2 for IESF and 3 for ISFF.

$$l_{m3}(s) = \frac{-0.5s^2-1.51s}{2.5s^2+8.55s+9.279}$$

The complementary sensitivity function of the simplified retail sector with nominal plant for each control scheme is written as followsw:

$$\eta_1(s) = \frac{1}{4s^2+2s+1}$$

: *for output feedback,*

$$\eta_{2,3}(s) = \frac{5.427s+1.189}{2s^4+7.04s^3+9.279s^2+5.427s+1.189}$$

: *for IESF and ISFF*

Substituting l_{mi} and $\eta_i(jw)$ into Eq. (10), robust stability conditions can be tested. Magnitudes of $|\eta_i(jw)\, l_{mi}|$ is seen in Fig. 6. It is seen that $|l_{mi}|^{-1}$ should be an upper-bound of $|\eta_i|$ for satisfying the robust stability condition. IESF control shows relative robustness compared to the other control schemes in the perturbed system. That means IESF control has a larger robust stability margin.

5.3 Performance robustness

If the condition (14) for disturbance rejection and the condition (20) for good command following are satisfied, then performance robustness has been guaranteed. One notes that performance robustness can be evaluated by the nominal ε_i, μ_i, η_i, α_i, β_{1i}, and l_{mi}. Based on the block diagrams of each control scheme (Fig. 2, Fig. 3, and Fig. 4), the return ratio α_i and feedforward loop β_{1i} are obtained by

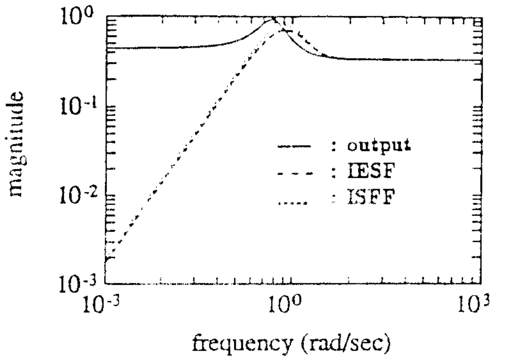
$$\alpha_1(s) = \frac{1}{2s(2s+1)},\ \alpha_2(s)$$

$$\alpha_3(s) = \frac{5.428s+1.189}{s^2(2s^2+7.04s+9.279)}$$

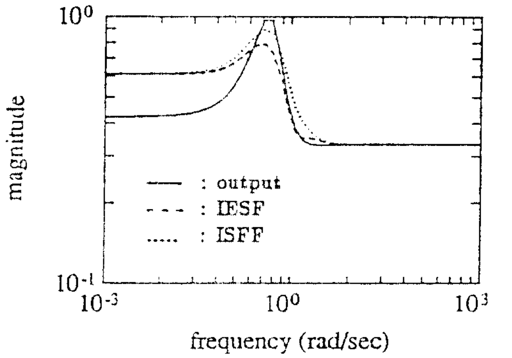
$$\beta_{11}(s) = \frac{5}{2s(2s+1)^2},\ \beta_{12}(s)$$

$$\beta_{13}(s) = \frac{3.566}{s^2(2s+1)(2s^2+7.04s+9.279)}$$

Substituting the corresponding nominal functions into Eqs. (14) and (20), one can obtain plots for robust performance as shown in Fig. 7(a) and (b). All three control laws satisfy the robust performance requirements of Eqs. (14) and (20) for given performance weights $w_r(jw)$ and $w_d(jw)$. This implies that all three controllers satisfy the robust stability condition. The reason is shown in Eqs. (14) and (20). Fig. 7(a) implies that the dynamic control (IESF or ISFF) has much better robust performance for disturbance rejection at low frequencies than the output feedback control. On the other hand, the output feedback control has better performance robustness for good command following at low frequencies as illustrated in Fig. 7(b). However, there is not much room for performance improvement in the output feedback control, since the peak magnitude of the *worst case* of the output feedback control is very close to unity. In other words, the dynamic control allows better performance weight over the entire frequency range. Hence for H_∞-analysis of the frequency domain, the maximum peak is significant to improve a desired performance.



(a) Robust disturbance rejection



(b) Robust good command following

Fig. 7 Robust performance illustration for output feedback, IESF and ISFF control law : (a) magnitude of $|\varepsilon_i w_d| + |\gamma_i l_{ii}|$ and (b) magnitude of $|w_t r| + |\eta_i l_{ii}|\left|1 + \left|\left(\frac{\rho h_i}{a_i} - k_h\right) w_r\right|\right|$

5.4 Time domain response

Based on the frequency domain analysis, the closed-loop responses to a step input are obtained for a nominal plant controlled by output feedback, IESF, and ISFF control laws. Actual inventory at retailer (IAR) is a output for a given example shown in Fig. 8. IESF and ISFF control laws give a more sluggish response for the inventory level than output feedback and have no overshoot. The undershoot at the starting region of the IAR response indicates the closed-loop zero in the right-half plane.

When the upper bound of delay at distributor (DUD) is 2.5 weeks, which is a nominal value 2 weeks, this value is compatible with the $\tilde{g}_1(s)$ in the frequency domain. Fig. 9 shows the step responses of IAR and UOD when the system has a uncertainty ΔDUD = 0.5week. IESF control gives the relatively better robust performance for good command following under the same design constraint. Actually, it can be shown that robust

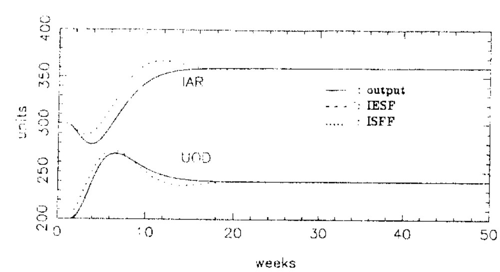


Fig. 8 Nominal time-domain responses for output feedback, IESF and ISFF law

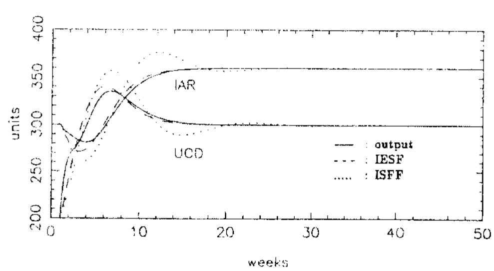


Fig. 9 Robust performance of time-domain responses for output feedback, IESF and ISFF control law with ΔDUD = 0.5week

performance is enhanced by applying the dynamic control scheme in the uncertain system when compared with the output feedback scheme. This result is congruous to the frequency domain analysis.

6. Conclusions

A design method of two dynamic controllers (IESF and ISFF) is proposed in the continuous-time domain. H_∞-analysis for stability and performance of the two-degree of freedom system are also considered in the frequency domain. If the perturbed systems satisfy robust performance specification, then robust stability and nominal performance are guaranteed. In the two degree of freedom configuration, the performance of good command following and disturbance rejection can be reached independently, Which means that the independence of the two performance indexes of the two-degree of freedom controller allows two independent performance weighting functions to be applied to design a controller. Numerical simulation results for the simplified retail sector which has a two-degree of freedom control structure, show that the dynamic control law has stability robustness and good performance robustness for disturbance rejection at low frequencies.

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